The average rate of change is a fundamental concept in our College Algebra course. We will cover the average rate of change from several different angles throughout this semester. For now, we will talk about understanding the average rate of change and how to calculate it.

Before looking at the formula, let’s look at the meaning of the average rate of change. This is something that we think about regularly. Suppose you drove 200 miles in four hours. Intuitively, you recognize that you drove 50 miles per hour on average. Even though we’ll add some formality and formulas, the idea behind the average rate of change is exactly this example.

The phrase I use to help students remember the average rate of change is “Change in the outputs divided by the change in the inputs.” Let’s do an example to see how this works.

This table defines a function B of t, where B is the cumulative domestic box office of the movie “Twisters”, measured in millions of dollars, and t is the number of days the movie is in theaters. Let’s calculate the average rate of change from t equals 1 to t equals 3. The change in the outputs is 81.3 minus 32.1 or 49.2. The change in the inputs is 3 minus 1 or 2. Dividing 49.2 by 2 gives 24.6. Thus, the cumulative domestic box office of “Twisters” increased by about 24.6 million dollars per day.

Notice that the units of the average rate of change have the word “per”. This is a hint that the number measures a rate of change. In general, the units for a rate of change are the units of the output “per” the units of the inputs.

The formula for the average rate of change of a function f from the inputs x sub 1 to x sub 2 is f of x sub 1 minus f of x sub 2 divided by x sub 1 minus x sub 2. You do not need to be precise on which number is x sub 1 or x sub 2. If you are consistent, the signs will take care of themselves.

Let’s use the formula to calculate the average rate of change from t equals 4 to t equals 7. Using the formula tells us we need to calculate B of 7 minus B of 4 over 7 minus 4. Substituting the values, we now need to calculate 119.6 minus 90.5 divided by 3. Finishing the calculation gives 9.7. So, we can say that “Twisters” earned about 9.7 million dollars per day on days 5, 6, and 7 of its theatrical run.

To finish, we’ll do an example with functions defined by a formula. Let f of x be 1 over x minus 2x. Calculate the average rate of change between x equals 2 and x equals 7. First, we’ll set up the average rate of change formula. Calculate f of 7 minus f of 2 divided by 7 minus 2. f of 2 is -3.5. f of 7 is -13.86 when rounded to two decimal places. -13.86 minus -3.5 is -10.36. Finally, divide by 5 to get -2.07 after rounding.

The average rate of change can be a negative number. We’ll talk more about that soon.